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Abstract— The Scarcity of renewable energy is one of the world issues that must be tackled. Moreover, the necessity of environmentally friendliness cooler is indispensable nowadays. There is an alternative technology which can handle the issues. Using thermoacoustic engine the waste heat and solar energy can be converted into acoustic energy that can be used for thermoacoustic cooler which is safe for environment. The aim of the study is to obtain the influence of engine stack/regenerator length on the efficiency of a heat-driven thermoacoustic refrigerator. The investigation was conducted numerically to find the optimum value of the regenerator length. It was found that the regenerator length is 0.014. Furthermore, the engine and cooler efficiency are 52% and 36% of the Carnot efficiency. In addition, the lowest heating temperature of the engine is 241°C which is good for exhaust heat recovery and green energy.

Keywords— heat-driven, machine, refrigerator

I. INTRODUCTION

Nowadays the lack of renewable energy is becoming the urgent issues all over the world [1]. Therefore, it is essential to tackle the problem. It can be handled using thermoacoustic technology. We could convert waste heat, geothermal and solar energy into sound energy by using thermoacoustic machine [2]. The engine can be used for driving cooler or electric power generation [3-6]. In other words, the acoustic power can be converted into electrical power [7].

On the other hand, the environmental issues such as depletion ozone layer and global warming potential are also need to be handled [8]. Thermoacoustic cooler technology can be a solution for it. In thermoacoustic, there is a cooler driven by an electrodynamic loudspeaker done by Farikhah et.al. They found that there is a decrease of the cooling temperature of 5°C [9]. However, the cooler relies on the non-renewable energy, so it is essential to use the renewable energy for example exhaust heat or sunlight energy. The device for converting the energy is called thermoacoustic engine. The energy conversion can be calculated using the thermoacoustic theory. Ueda in 2016 numerically calculated the energy conversion in the regenerator screen [10]. Some researchers also studied about the engine and analyzed the heating temperature and the performance. The numerical study was conducted by Utami et.al. They focused on the effect of the radius of the engine stack to the performance of thermoacoustic engine. Rokhmawati in 2020 investigated the mean pressure and pipe’s radius impact on the efficiency of the entire device. Furthermore, Farikhah in 2021 numerically studied the influence of stack length on the performance of thermoacoustic engine. However, this is without the load such as a thermoacoustic cooler [11-13]. The engine can be combined with the cooler. In other words, thermoacoustic cooler can be driven by the thermoacoustic machine. In 2017 there is a study on the performance of a heat-driven thermoacoustic cooler with different porosity [4]. In 2017 Farikhah studied the efficiency of a thermoacoustic system with machine and refrigerator stack in a pipe [5]. The influence of mean pressure on the efficiency of a heat-driven thermoacoustic chiller was studied in 2020 [6]. However, the heating temperature of the machine is still high [4-6]. Therefore, it is important to find the low heating temperature for waste heat recovery which is beneficial for renewable energy. To obtain the low heating temperature of the machine, the multistage engine must be set up. There is an investigation the influence of stack radii on the efficiency of multistage machine [14]. However, in this case we need to installed 4 regenerators which is more complicated. Therefore, designing the simple and efficient device with low heating temperature such as single stage cooler driven by engine is essential due to the simplicity. In this case, the dimensionless engine regenerator length $L_e / L_{loop}$ is the parameter. Thus, in this investigation we focus on finding the impact of engine stack length on the performance of the cooler driven by the machine. We focus on the efficiency of the whole system because it
means that we focus on decreasing the dissipation. Thus, in the future it will be good for commercial cooler industry.

II. METHOD

A. Calculation Model

The Design of a heat-driven thermoacoustic cooler is presented in Fig. 1. It comprises of two stacks, four heat exchangers and two thermal buffer tubes which is filled with helium and the pressure is 3 MPa. The looped tube length is 280 cm and the diameter is 4 cm. The first regenerator is employed as an engine and the second regenerator is employed as a cooler. The engine is located between the ambient and hot heat exchanger. In the vicinity of the heat exchanger is set a thermal buffer tube to allow the temperature change gradually from heating temperature to ambient temperature. The cooler is placed between the ambient and cold heat exchanger. Another thermal buffer tube is put next to the cold heat exchanger allowing the cooling temperature increasing gradually to the ambient temperature. The length of the engine regenerator is varied between 30 mm to 90 mm which are set as the parameter, while the cooler length is set as a constant value at 40 mm. In addition, the narrow channel radius of the machine and refrigerator stack are 0.1 mm and 0.5 mm, respectively.

B. Calculation Method

In the simulation, we use these two equations who is derived by Rott shown in eq. 1 and eq. 2. [16]. The axial coordinate along the pipe is expressed as \( x \) where the initial point is set at the engine regenerator ambient side. \( \delta, \chi_v, \) and \( \chi_a \) are the thermal penetration depth and the thermoacoustic function. The mean density, specific heat ratio and the Prandtl number of Helium are denoted as \( \rho, \gamma \) and \( \sigma \), respectively. The pressure and velocity are denoted as \( P \) and \( U \), respectively.

If \( dT_m/dx = 0 \), eq. (1) and eq. (2) can be solved analytically, but if \( dT_m/dx \neq 0 \), the two equations have to be computationally integrated [15]. We used two temperature conditions. The constant value of \( dT_m/dm \) is the first condition and the calculated value of \( dT_m/dm \) is the second conditions. Here, the constant enthalpy flow along the regenerator is set. It is denoted as \( H \) [see eq. 3]

\[
\frac{dP_1}{dx} = -\frac{i\omega \rho m}{1 - \chi_v} U_1 \\
\frac{dU_1}{dx} = -\frac{i\omega [1 + (\gamma - 1)\chi_a]}{\gamma P_m} + \frac{\chi_a - \chi_v}{(1 - \chi_v)(1 - \sigma)} \frac{1}{T_m} \frac{dP_m}{dx} U_1
\]

\[
H = \dot{W} - \dot{Q}
\]

As eq. 1 and 2 is substituted into eq. 3, so the equation become as follow

<table>
<thead>
<tr>
<th>The geometrical Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine ambient heat exchanger length</td>
<td>10 mm</td>
</tr>
<tr>
<td>Engine ambient heat exchanger radius</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Engine ambient heat exchanger porosity</td>
<td>0.5</td>
</tr>
<tr>
<td>Engine Regenerator/stack length</td>
<td>30-90  mm</td>
</tr>
<tr>
<td>Engine Regenerator/stack porosity</td>
<td>0.77</td>
</tr>
<tr>
<td>Engine regenerator/stack flow channel radius</td>
<td>0.1 mm</td>
</tr>
</tbody>
</table>
Using eqs. 1-4, temperature gradient along the regenerators can be calculated if the boundary conditions of the oscillatory pressure \( P \) and Velocity \( U \) are given. In this calculation, the thermal conduction and the acoustic streaming are neglected [17,18,19]. In this simulation the heating temperature \( T_h \), angular frequency and impedance \( P/U \) are calculated using the stability limit calculation [15].

Equation 1 and 2 are converted in a matrix form.

\[
C(x) = \begin{pmatrix} 0 & -i\omega\rho_m & 1 \end{pmatrix} \begin{pmatrix} -i\omega[1 + (\gamma - 1)\chi_a] & \chi_a - \chi_x & 1 \end{pmatrix} \frac{dt_m}{d\chi_m}
\]

\[
C(x) \begin{pmatrix} P_1(x + \Delta x, t) \\ U_1(x + \Delta x, t) \end{pmatrix} = \begin{pmatrix} P_1(x, t) \\ U_1(x, t) \end{pmatrix}
\]

\[
C' = \frac{1}{6}(RK_1 + 2RK_2 + 2RK_3 + RK_4)
\]

Here, we used the Fourth-order Rung Kutta Method

\[
RK_1 = C(x)
\]

\[
RK_2 = C \left( x + \frac{\Delta x}{2} \right) \left( E + \frac{\Delta x}{2}RK_1 \right)
\]

\[
RK_3 = C \left( x + \frac{\Delta x}{2} \right) \left( E + \frac{\Delta x}{2}RK_2 \right)
\]

\[
RK_4 = C(x + \Delta x) \left( E + \Delta xRK_3 \right)
\]

The unit matrix is denoted as E

\[
\begin{pmatrix} P_1(x,t) \\ U_1(x,t) \end{pmatrix} = M_l(x,x_0) \begin{pmatrix} P_0(x_0,t) \\ U_0(x_0,t) \end{pmatrix}
\]

\[
M_l(x,x_0) \equiv \begin{pmatrix} (E + \Delta xC'_n)(E + \Delta xC'_n) & \ldots \end{pmatrix}
\]

\[
Y_{d,e} = \begin{pmatrix} 1 & 0 \\ 0 & A_d/A_e \end{pmatrix}
\]

There are 10 components in a heat-driven thermoacoustic refrigerator as mentioned in table. 1. The total path area is denoted as \( A \). The total system transfer matrix is expressed as:

\[
M_{tot} = M_{10}O_{10}M_{09}O_{09}M_{a}O_{a}M_{7}O_{7}M_{6}O_{6}M_{5}O_{5}M_{4}O_{4}M_{3}O_{3}M_{2}O_{2}M_{1}
\]

\[
M_{tot} \begin{pmatrix} p_{ea} \\ U_{ea} \end{pmatrix} = \begin{pmatrix} p_{ea} \\ U_{ea} \end{pmatrix}
\]

If the determinant of the matrix \( (M_{tot} - E) \) is zero, the solution \( p_{ea},U_{ea} \) of equation 15 is nonzero

\[
(m_{11} - 1)(m_{22} - 1) - m_{12}m_{21} = 0
\]

The element of \( M_{tot} \) is denoted as \( m_{ij} \) so equation 15 was solved to obtain the stability limit conditions that shows the gas spontaneously oscillates.

The total efficiency of the whole thermoacoustic refrigerator system can be expressed as:

\[
\eta_{all} = \eta_{2,e} \eta_{2,c} \eta_{tube}
\]

Where \( \eta_{all,2} \) is the entire efficiency in the whole system. \( \eta_{all,2} \) is divided into three efficiencies; \( \eta_{2,e} \) is the second law efficiency of the engine, \( \eta_{2,c} \) is the second law of the cooler and \( \eta_{tube} \) is the efficiency of the pipe.

\[
\eta_{2,e} = \eta_e / \eta_{Carnot}
\]

\( \eta_{2,e} \) is defined as eq. 17, thermal and Carnot efficiency are denoted as \( \eta_e \) and \( \eta_{Carnot} \).

\[
\eta_{tube} = \eta_{tube} / \Delta W_e
\]

\( \eta_{2,c} \) is the ratio of acoustic power absorbed by refrigerator to the acoustic power generated by the machine.

III. RESULTS AND DISCUSSIONS

A. Performance of the whole System

Figure 2 shows the validation between the numerical calculation and the experimental one. As we can see, the pressure amplitude \( |P| \), Velocity \( |U| \) and the phase different \( \phi \) have a good agreement between the calculation and the experimental work.

The entire performance of the cooler system \( \eta_{2,all} \) is provided in Fig. 3. The performance comprises of efficiency of machine, tube and cooler. In this simulation, the regenerator length \( L_s \) is varied from 3 cm to 9 cm and the loop tube length \( L_{loop} \) is 280 cm. In this calculation, we used the dimensionless
ratio of the stack and the pipe lengths as the parameter denoted as $L_s/L_{loop}$. As we can see in fig. 3, when $L_s/L_{loop}$ rise from 0.010 to 0.014, $\eta_{all}$ increase from 3.0% to 4.0% of Carnot efficiency. Then, when $L_s/L_{loop}$ increase from 0.014 to 0.30 $\eta_{all}$ drop from 4% to 3% of Carnot efficiency. According to eq.16, the whole efficiency of the system depends on the second law efficiency of the engine $\eta_{2,e}$, cooler $\eta_{2,c}$ and tube $\eta_{tube}$, so it is essential to reveal those efficiency.

As mentioned above, the superior total performance of the device is achieved when the length is 0.014. According to figure 4, the superior efficiency of the device is due to the high value of all components. The efficiency of the engine, cooler and tube are 52%, 32% and 21% of the upper limit value.

As we can see in Figure 4, $\eta_{2,e}$ has the highest value compare to the $\eta_{2,c}$ and $\eta_{tube}$. Furthermore, as $L_s/L_{loop}$ go up from 0.010 to 0.030, $\eta_{2,e}$ rise from 50% to 56% of the upper limit values. On the other hand, $\eta_{2,c}$ drop from 39% to 15% when $L_s/L_{loop}$ increase, while $\eta_{tube}$ increase from 16 to 36% when $L_s/L_{loop}$ rise from 0.010 to 0.030.

Based on eq. 20, the second law efficiency of the refrigerator depends on $COP$ and $COP_{Carnot}$. As we can see in the fig. 5, the high value of second law efficiency of the refrigerator is because of the high value of $COP$. When $L_s/L_{loop}$ is 0.014, COP is 3.5. the highest value is when the length is 0.010 at 3.8, but the superior entire cooler system is at 0.014. It is because the engine and tube efficiency at 0.014 are higher than that of at 0.010 [see figure 4]. Therefore, in this study, we focus on the value of $COP$ especially at 0.014. Moreover, the $COP_{Carnot}$ are set at 9.75 since the cooling and ambient temperature are set at 0 °C and 28 °C.
\( \text{COP} \) is the ratio of the cooling power \( Q_c \) to the acoustic power consumed by the cooler \( \Delta W_c \) as we can see in eq. 21. Therefore, the high value of \( \text{COP} \) can be attributed to the high value of cooling power \( Q_c \) as shown in Fig. 6. It shows the highest cooling power when the length is 0.014. It was found that the highest cooling power is 0.23 W. The cooling power is the summation between three different power that can be expressed as follows [21].

\[
Q_c = Q_{\text{prog}} + Q_{\text{stand}} + Q_d \quad (24)
\]

Figure 7 shows the dimensionless parameter of \( Q_{\text{prog}} \), \( Q_{\text{stand}} \) and \( Q_d \) to \( Q_c \). \( Q_{\text{prog}}/Q_c \) implies the travelling wave [20]. Figure 8 shows \( Q_{\text{stand}}/Q_c \) implies the standing wave and figure 9 shows \( Q_d/Q_c \) implies the dream pipe [21]. Compare to \( Q_{\text{stand}} \) and \( Q_d \), \( Q_{\text{prog}} \) is the power which has high contributions on the high performance due to the reversible energy conversion which means less dissipation. As can be seen in figure 7, the highest \( Q_{\text{prog}}/Q_c \) is achieved when the length ratio \( L_z/L_{\text{loop}} \) is 0.010 followed by 0.014. In fig. 8 the highest \( Q_{\text{stand}}/Q_c \) is reached when the length ratio is 0.010. As \( Q_{\text{stand}} \) implies the standing wave, so at this length the irreversible energy conversion occurs. Eventhough \( Q_{\text{prog}}/Q_c \) is highest at 0.010, but \( Q_{\text{stand}}/Q_c \) is high as well. It indicates that at 0.010, the dissipation occurs more than that at 0.014. As a result, the most efficient would achieved when the length is 0.014.

\( Q_{\text{prog}}/Q_c \) and \( Q_{\text{stand}}/Q_c \) shown in Fig. 8 and 9 have positive values. It indicates that the flow directions of the cooling energy are the same. However, \( Q_d/Q_c \) has negative values. It means that the flow direction of the cooling energy is the opposite direction. As a consequence, when \( Q_d \) is high the total \( Q_c \) decrease. Since at 0.010 the value of \( Q_d/Q_c \) is the highest one, so the total \( Q_c \) will decrease at \( L_z/L_{\text{loop}} = 0.010 \). Thus, the most efficient is when the \( L_z/L_{\text{loop}} \) is 0.014 due to the high value of \( Q_{\text{prog}} \) while \( Q_{\text{stand}} \) and \( Q_d \) are lower than that at 0.010.

B. The Heating Temperature

Figure 10 shows the engine regenerator length \( L_z/L_{\text{loop}} \) as a function of heating temperature \( T_h \). The heating temperature increases from 241 °C to 386 °C. At 0.010 the heating temperature is the lowest which can be used for waste heat recovery and renewable energy for generating the engine.
The influence of regenerator length on the efficiency of a heat-driven thermoacoustic refrigerator was investigated. It was found that when the dimensionless engine regenerator length is 0.014, the efficiency of the whole device is superior. The finding is essential for guidance in experimental work. It means when ratio of the stack length to the loop length is 0.014, the efficiency of the whole system is 4% of the Carnot efficiency and the efficiency of the engine, cooler and tube are 52% and 36% and 21%, respectively. Moreover, the lowest heating temperature of the engine was found 241°C at the length ratio 0.010. It means that the heating temperature can be used for exhaust heat recovery and green energy.

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