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Dynamic stability response in micro-beams assuming porosity based on numerical solution

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***Abstract*-The dynamic stability response of micro functionally graded materials (FGM) porous beam is studied. The structural damping is expected using Kelvin-Voigt theory. The microbeam is placed on the viscoelastic foundation with spring, shear and damper constants. The size influences are expected based on the couple stress theory with one length scale material factor. The Timoshenko theory for microbeam is employed for the governing final equation on the basis of Hamilton's principle. The final motion couled equations are attained by differential quadrature method (DQM) for calculating the dynamic stability area. The influences of various components of FG index, porosity, geometric and structural components for the microbeam on the dynamic response of the structure are exposed. It is obvious that with enhancing the porosity value, the dynamic instabillity region (DIR) shifts to higher frequencies.**

Keywords:* *Dynamic stability; Micro beam; FGM; porosity; Numerical method.

I. INTRODUCTION

The smart piezoelectric structures with various solicitations such as actuators, sensors and seismograph are attentive between researchers. This smart piezoelectric material may be applied for smart control of the dynamical structures. Yang and Shen [1] examined the dynamic buckling of circular laminated cylindrical shells covered by smart layer with simple supported under the axial excitation load. Mendes et al. [2] investigated the application of a smart lauers for energy harvesting from in smart rods subjected to normal force. Farrokhian [3] studied vibration response of nanocomposite smart plate.

About the modelling of structures, there are many works such as Yang and He [4] studied vibration and buckling analysis of an functionally graded (FG) orthotropic microplate using modified couple stress model. Kim and et al. [5] examined buckling, vibration and bending response of porous FG microplates using thick and thin plate theories. Farokhian and salmani- Tehrani [6] calculated frequency of advanced smart sandwich carbon nanotube with sensor layers. Dynamic and static deflection of the FG porous smart sandwich plate on the Kerr medium under electric and thermal loads were presented by Kumar and Harsha [7].

According to above mentioned papers, the dynamic response of the porous FG microbeam is untaken. Using Hamilton's principle and Timoshenko theory of beam, the governing final equations are solved by DQM. The effects of structural damping, FG index, geometric parameters and porosity are signposted on the DIR of the microstructure.

II. FORMULATION

Fig. 1 displays a micro FGM beam with porosity. The length of beam is L and the thickness is h . The Visco-elastic foundation is supposed by damper, spring and shear elements.

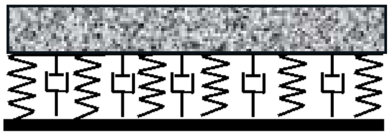

Fig. 1. A micro beam with porosity

The displacements can be assumed as [8]:

$$U(x,z,t)=u_0(x,t)+z\,\alpha(x,t), \tag{1}$$

$$V(x,z,t)=0, \tag{2}$$

$$W(x,z,t)=w_0(x,t), \tag{3}$$

where respectively, u_0, w_0 indicate the mid-plane deflections in the *x* and *z* directions. The normal and shear strains are:

$$\varepsilon_{xx}=\frac{\partial u_0}{\partial x}+z\,(\frac{\partial \alpha}{\partial x}), \tag{4}$$

$$\varepsilon_{xz}=\alpha+\frac{\partial w_0}{\partial x}, \tag{5}$$

The stress-strain relations are [9]:

$$\sigma_{ijt}=C_{ijkl}\varepsilon_{klt}, \tag{6}$$

where σ_{ijt} is stress tensor; C_{ijkl} are the elastic coefficients. Furthermore, since the structure is FG porous, we have:

$$C_{ij}=(C_{ceramic}-C_{metal})\left(\frac{z}{h_b}-\frac{1}{2}\right)^p+C_{metal}-(C_{ceramic}+C_{metal})\frac{\alpha_p}{2}, \tag{7}$$

where p=0 and inf, is shows a fully ceramic or metal, respectively. Supposing Kelvin-Voigt model, the elastic coefficients is assumed as:

$$C_{ij}=C_{ij}\left(1+g\frac{\partial}{\partial t}\right), \tag{8}$$

where g is the damping structural coefficient. In addition, the potential energy using the couple stress theory may be written as:

$$U=(\frac{1}{2}\int(\sigma_{ij}\varepsilon_{ij}+m_{ij}\chi_{ij})dv \tag{9}$$

where

$$\chi_{ij}=\frac{1}{2}(\varphi_{i,j}+\varphi_{i,j}), \tag{10}$$

$$\phi_i=\frac{1}{2}(curl\,(u))_i, \tag{11}$$

$$m_{ij}=2\mu I_0^2\chi_{ij}, \tag{12}$$

where ϕ_i and γ_i are the infinitesimal rotation and dilatation gradient; I_0 is material length scale parameter. The force of visco Pasternak medium is:

$$q=k_wW+c_d\dot{W}-k_g\nabla^2W, \tag{13}$$

where k_w, c_d and k_g are the spring, damper and shear constants. The external work is as:

$$w=\int q\,dA, \tag{14}$$

The kinetic energy is:

$$k_t=\int\int\left[I_0\left(\frac{\partial u_0}{\partial t}\right)^2+I_2\left(\frac{\partial \alpha}{\partial t}\right)+I_0\left(\frac{\partial w_0}{\partial t}\right)^2\right]dV, \tag{15}$$

where

$$\left(I_0,I_2\right)=\int\left(1,z^2\right)\rho(z)dA, \tag{16}$$

Utilizing Hamilton's principle, we have:

$$\begin{aligned}\delta u_0:&\frac{\partial}{\partial x}\left(A_{11}\frac{\partial u_0}{\partial x}+P_{11}\frac{\partial \alpha}{\partial x}\right)+-\frac{\partial^2}{\partial x^2}\left(A_\gamma\frac{\partial^2 u_0}{\partial x^2}+P_\gamma\frac{\partial^2\alpha}{\partial x^2}\right)-I_0^t\frac{\partial^2 u_0}{\partial t^2}\\&+\frac{2}{75}\frac{\partial^2}{\partial x^2}\left(-3A_\eta^t\frac{\partial^2 u_0}{\partial x^2}-3P_\eta\frac{\partial^2\alpha}{\partial x^2}\right)+\frac{2}{25}\frac{\partial^2}{\partial x^2}\left(2A_\eta\frac{\partial^2 u_{0t}}{\partial x^2}+2P_\eta\frac{\partial^2\alpha}{\partial x^2}\right)+\\&\frac{3}{75}\frac{\partial^2}{\partial x^2}\left(-3A_\eta\frac{\partial^2 u_{0t}}{\partial x^2}-3P_\eta\frac{\partial^2\alpha}{\partial x^2}\right)-\frac{1}{75}\frac{\partial^2}{\partial x^2}\left(3A_\eta^t\frac{\partial^2 u_0}{\partial x^2}+3P_\eta\frac{\partial^2\alpha}{\partial x^2}\right)=0,\end{aligned} \tag{17}$$

$$\begin{aligned}\delta w_0:&I_2\frac{\partial^2\alpha}{\partial t^2}+-\frac{1}{25}\frac{\partial^2}{\partial x^2}\left(-3H_\eta\frac{\partial\alpha}{\partial x}\right)-\frac{1}{75}\frac{\partial^3}{\partial x\,3}+\left(3B_\eta\frac{\partial^2 u_0}{\partial x^2}+3F_\eta\frac{\partial^2\alpha}{\partial x^2}\right)\\&+\frac{1}{225}\frac{\partial^2}{\partial x^2}\left(3H_\eta\frac{\partial\alpha}{\partial x}\right)-\frac{2}{225}\frac{\partial^2}{\partial x^2}\left(-3H_\eta\frac{\partial\alpha}{\partial x}\right)+\frac{\partial^2}{\partial x^2}\left(Q_\gamma\frac{\partial\alpha}{\partial x}\right)-\frac{4}{25}\frac{\partial^3}{\partial x\,3}\\&\left(B_\eta\frac{\partial^2 u_0}{\partial x^2}+F_\eta\frac{\partial^2\alpha}{\partial x^2}\right)+k_g\frac{\partial^2 w_0}{\partial x^2}-Kw_0-C_d(\frac{\partial}{\partial t}w_0)=0,\end{aligned} \tag{18}$$

$$\begin{aligned}\delta\alpha:&\frac{2}{5}C_\eta\frac{\partial^2 u_0}{\partial x^2}-\frac{2}{75}\frac{\partial}{\partial x}\left(-3O_\eta\frac{\partial\alpha}{\partial x}\right)-I_2\left(\frac{\partial^2\alpha}{\partial t^2}\right)-\frac{3}{25}\frac{\partial}{\partial x}\left(-3O_\eta\frac{\partial\alpha}{\partial x}\right)+\\&+\frac{3}{75}\frac{\partial^3}{\partial x\,3}\left(-3P_\eta\frac{\partial^2 u_0}{\partial x^2}\right)-\frac{1}{75}\frac{\partial^2}{\partial x^2}\left(3P_\eta\frac{\partial^2 u_0}{\partial x^2}-+3D_\eta\frac{\partial^2\alpha}{\partial x^2}\right)=0,\end{aligned} \tag{19}$$

where

$$A_\gamma,B_\gamma,R_\gamma=\int_{-h/2}^{h/2}2\left(1,z,z^2\right)\mu_t I^2{}_0dz \tag{20}$$

III. SOLVING METHOD

In DQ method, we have the basic below relation [10,11]:

$$\frac{d^n f_x(x_i)}{dx^n}=\sum_{k=1}^{N_x}A_{ik}^{(n)}f(x_k),\qquad n=1,\ldots,N-1. \tag{21}$$

where

$$X_i=\frac{L}{2}\left[1-\cos\left(\frac{i-1}{N_x-1}\right)\pi\right],\qquad i=1,\ldots,N_x \tag{22}$$

$$A_{ij}^{(1)}=\begin{cases}\dfrac{M(x_i)}{(x_i-x_j)M(x_j)} & for\quad i\neq j,\quad i,j=1,2,\ldots,N_x\\ -\sum\limits_{\substack{j=1\\ i\neq j}}^{N_x}A_{ij}^{(1)} & for\quad i=j,\quad i,j=1,2,\ldots,N_x\end{cases} \tag{23}$$

where

$$M(x_i)=\prod_{\substack{j=1\\ j\neq i}}^{N_x}(x_i-x_j), \tag{24}$$

The final form of equations based on Bolotin's method are:

$$\left|\left([K]-\left(\kappa\pm\frac{\beta}{2}\right)\bar{P}_{cr}[K]_G\right)\pm[C]\frac{\omega}{2}-[M]\frac{\omega^2}{4}\right|=0, \tag{25}$$

in which $[K]$, $[C]$ and $[M]$ respectively, are matrices stiffness, damp and the mass matrixes, $[K_G]$ is the geometric hardness matrix.

IV. RESULTS AND DISCUSSION

Here, an FG porous microbeam is expected made from metal with elastic modulus of 73 GPa and Ceramic with elastic modulus of 400 GPa.

For comparing the results of this paper with vibration of FG porous piezoelectric beam, the structure is simplified by a FG porous beam with piezoelectric layers based on FSDBT. Assuming the geometrical and material proprieties the same as Zia et al. [12], the frequency of the microstructure is compared with Zia et al. [12] in Table 1. It is found that, the outcomes of this article are in close to Zia et al. [12] and the little difference is due to this fact the in this paper, we assumed the linear strain relations while in Zia et al. [12], the nonlinear strain relations are considered.

TABLE I. Validation of this article with Zia et al. 2018 for vibration of FG porous piezoelectric beam

α_p	V_0 (volt)	FG index			
		0.5	1	5	10
0	+0.05, Zia et al. [12]	1.37	1.21	0.92	0.84
	+0.05, Present work	1.40	1.25	0.97	0.87
	0, Zia et al. [12]	1.52	1.39	1.17	1.11
	0, Present work	1.58	1.43	1.20	1.14
	-0.05, Zia et al. [12]	1.66	1.54	1.36	1.32
	-0.05, Present work	1.71	1.58	1.39	1.3
0.1	+0.05, Zia et al. [12]	1.36	1.16	0.79	0.69
	+0.05, Present work	1.42	1.21	0.83	0.72
	0, Zia et al. [12]	1.52	1.36	1.1	1.04
	0, Present work	1.59	1.42	1.15	1.07
	-0.05, Zia et al. [12]	1.67	1.54	1.33	1.28
	-0.05, Present work	1.73	1.58	1.36	1.30

Before presenting the results of this paper, the convergence of the DQM is studied in Fig. 2. In this figure, the non-dimentional frequency versus the DQ points is illustrate in this figure. That abuse that, the non-dimentional frequency is reduced with enhancing the DQ points until N=17 which the outcomes become converge and we have not any change in the frequency.

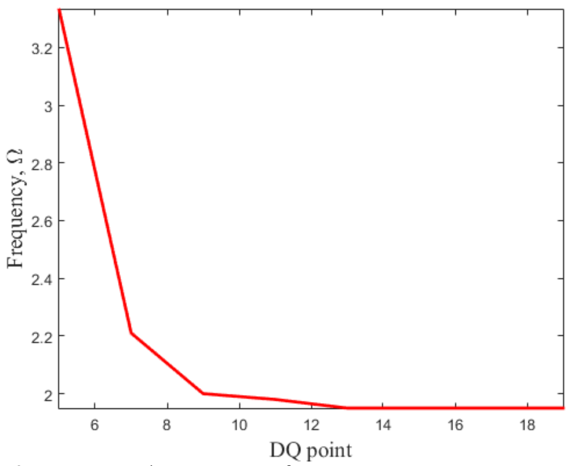

Fig. 2. Accuracy and convergence of DQM

The effect of damping parameter on the DIR is showed in Fig. 3. It is obvious that according to the damping, the frequency is reduced and DIR is happening at lower frequency.

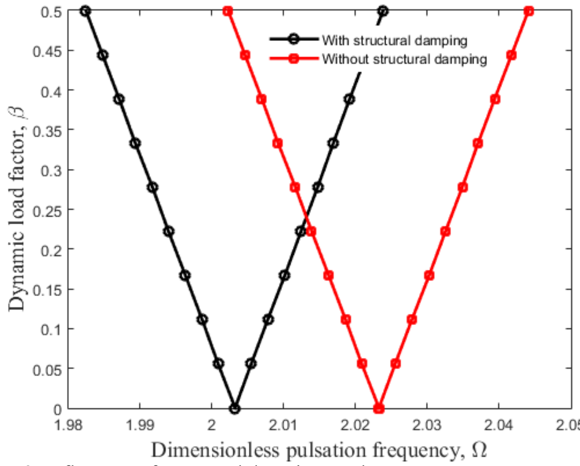

Fig. 3. Influences of structural damping on the DIR

Fig. 4 demonstrates the effect of porosity charactecrtic on the DIR. As can be found, the porosity can decrease the frequency due to reduction of stiffness. furthermore, enhancing the porosity, shifts the DIR to lower frequencies.

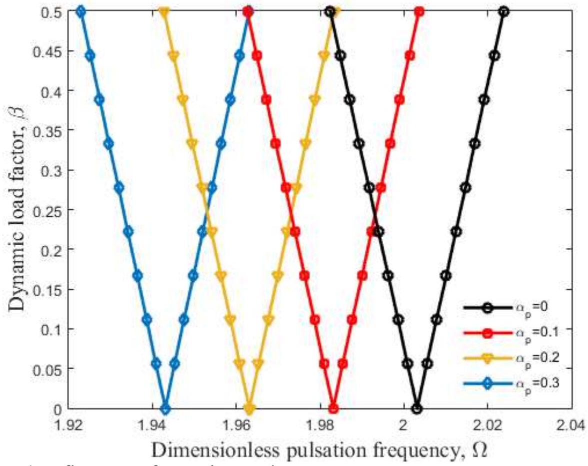

Fig. 4. Influences of porosity on the DIR

Fig. 5 determine the influence of FG index on the DIR of the FG porous microbeam. It is found that with increasing FG index, the DIR shifts to left. It is due to this reason that with enhancing the FG index, the stiffness is reduced.

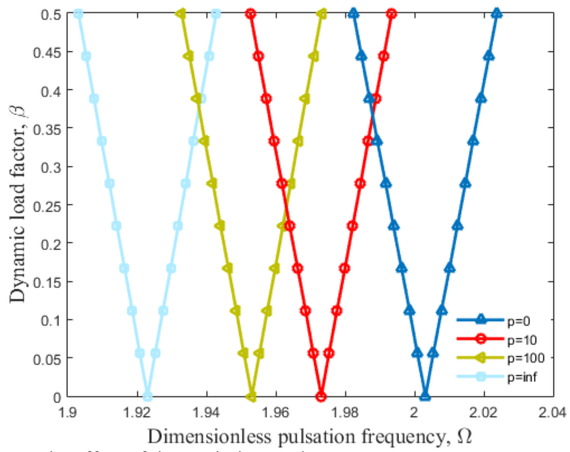

Fig. 5. The effect of the FG index on the DIR

The influence of size parameter on the DIR is exposed in Fig. 6. It is obvious that with enhancing the size parameter, the frequency is improved and DIR shifts to higher frequency.

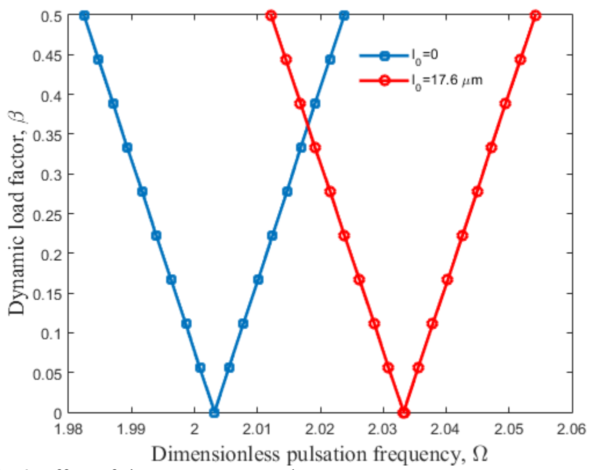

Fig 6. Effect of size parameters on the DIR

V. CONCLUSIONS

The dynamic response of viscoelastic porous microbeam was studied using Kelvin-Voigt model. The beam was FGM. The structural was modelled by Timoshenko beam model. Applying DQM and Bolotin's method, the motion final equations were solved. The effects of porosity, FG index, damping and geometric factors were examined on the dyanmic response. As can be found, the porosity can reduce the frequency due to reduction of stiffness. It was obvious that assuming the damping, the frequency was reduced and DIR was happening at lower frequency. It was found that with increasing FG index, the DIR shifts to left. It was obvious that with enhancing the size parameter, the frequency was improved and DIR shifts to higher frequency.

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